

FRANKLIN MATH BOWL

Algebra I

Problem Solving

2000

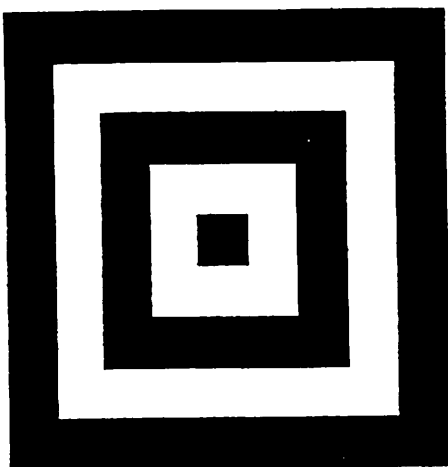
Each team must have ONE complete write-up for each problem. Explain your reasoning. The problem will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

- I. Numbers of the form $\frac{n(3n-1)}{2}$ are called pentagonal numbers.

List the first 5 pentagonal numbers.

Take each pentagonal number and divide it by 9 and record the remainder. This remainder is called the digital root of the pentagonal number. How many different digital roots are there for pentagonal numbers?

- II. When the Smiths remodeled their house, they tore out the walls of their downstairs bathroom. This left the wall above the old tub covered with ceramic tile as part of the new family room. The pattern was interesting, so they decided to put a frame around it and use it for a dart board and use darts with suction cup tips. The “dart board” is pictured below.



The wall is covered with square tiles and there is one black tile in the center of the wall. Lana, the oldest of the Smith children, assigns points to each “ring” of tiles so that the points value of a “ring” times the area in that ring will equal the point value of any other “ring” times the area of that other ring. What are the smallest values that she can assign so that all values are integers?

III. In old movies the wheels of a wagon appeared to turn backward. This was because one saw the spokes of the wheel in each frame. Let's say that the wheel was turning clockwise. Furthermore, assume that there were spokes at 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11. The spoke which was at 1 in the first frame moves to a point just before the position that was originally occupied by a spoke at 2. When the next frame is shot it appears that the frame at 2 had moved counterclockwise a small amount instead of having moved clockwise a greater distance.

1. If the wheel had a radius of 2 feet, how far would the wagon move when it made one complete revolution?
2. The wagon wheel had 12 spokes evenly spaced around the wheel. If the wheel turned at a rate so that in each frame of the film every spoke has advanced to the location which was held by the adjacent spoke in the last frame, how far has the wagon moved in the time between frames?
3. If 16 frames were shot per second and each spoke advanced $\frac{5}{6}$ of the way to the next spoke in each frame, the wheels would appear to be turning backward. However, they are turning forward. How fast was the wagon going if it had spokes of length 2 feet? Express the speed in feet per minute.